Module - 6

Used a control volume analysis to find the drag force on the plate. He invented momentum integral equation theory. (for incompressible boundary layer).

Boundary layer.

The boundary layer is the thin region of flow adjacent to surface, where the flow is retarded by the friction influence of friction between a solid surface and fluid.

* for a laminar flow, the boundary layer thickness is predicted by Blasius
* for a turbulent flow, the boundary layer thickness is predicted by Prandtl.

⇒ Prandtl states that " The condition of zero fluid velocity at the solid surface is referred to as no slip and the fluid b/w the surface & the free stream fluid is termed boundary layer.

Boundary layer theory.

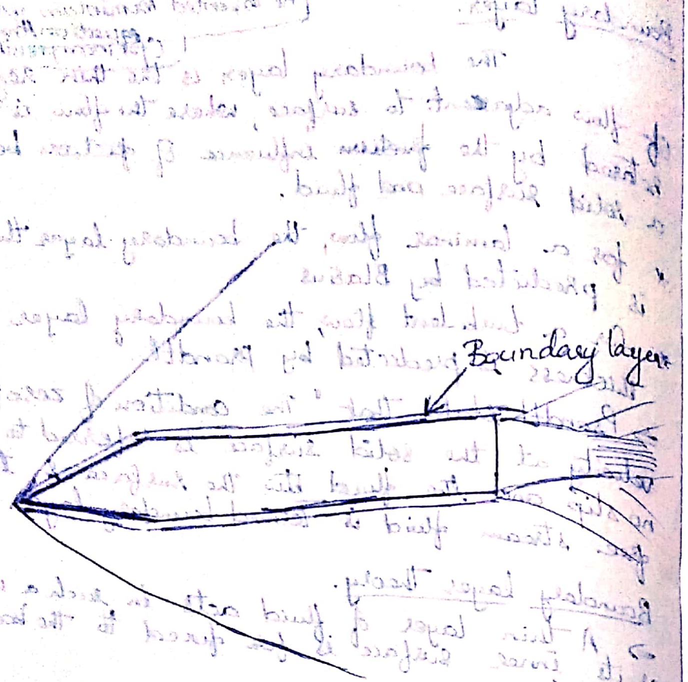
→ A thin layer of fluid acts in such a way, as its inner surface is fixed to the boundary of the body.

→ The velocity of flow at boundary layer is zero. The velocity of flow will be away from boundary layer. At the extreme layer the velocity is Max :-

→ when velocity increases, the Reynolds no :- will also increases.

$$\boxed{Re = \frac{\rho V D}{\mu}}$$

μ = Dynamic viscosity



The boundary layer on an aerodynamic body.

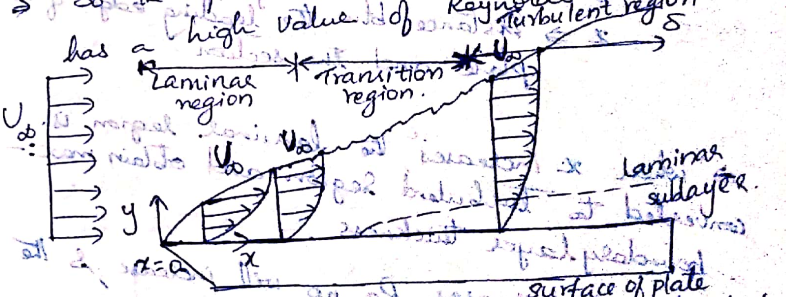
Introduction to boundary layers

Boundary layer Properties

a. Velocity and temperature profiles
b. Shear stress and heat Transfer.
c. Boundary layer thickness
d) Displacement thickness
e) Momentum thickness
f) Laminar and turbulent flow

The boundary layer equations

→ So the portion outside the boundary layer has a high value of Reynold's number.



Consider a thin flat plate. Let U_∞ is the velocity of flow over the flat plate.

i) In laminar region, the velocity of fluid is very low and velocity is varying parabolically.

ii) Transition region ⇒ Region btw laminar & turbulent region.

iii) Turbulent region ⇒ The velocity of fluid is more. In turbulent region, a laminar layer also exist & known as laminar sublayer.

Boundary layer Thickness (δ)

Boundary layer Thickness is depends upon Reynold's number

$$\boxed{Re = \frac{U_\infty x}{\nu}}$$

where U_∞ = velocity of fluid

ν ⇒ Kinematic viscosity of fluid

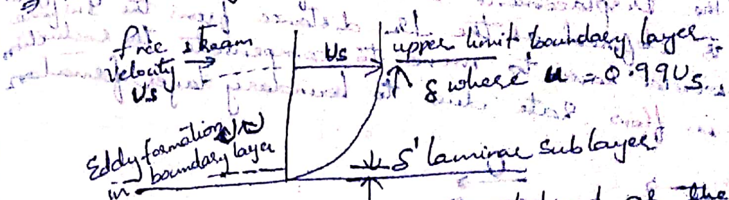
x ⇒ Distance btw the leading edge of the plate and the section

⇒ when x increases, the laminar region is converted to turbulent region and obtain max:- boundary layer thickness

⇒ when U_∞ increases, Re no will increase, & the flow became turbulent.

⇒ when viscosity increases, Re no:- decreases, ie: viscosity is more in laminar layer flow.

⇒ when density increases, Re no:- increases.



Boundary layer thickness is defined as the distance from the surface to the point where the local velocity = 99% of the stream velocity.

$$\boxed{\delta = y \,(u = 0.99U_s)}$$

(i) for laminar flow

According to Blassius

$$\boxed{\delta_{lam} = \frac{5x}{\sqrt{Re}}}$$

ie, the thickness of boundary layer (δ) is equal to zero at the leading edge ($\because x = 0$) and increases to the trailing edge.

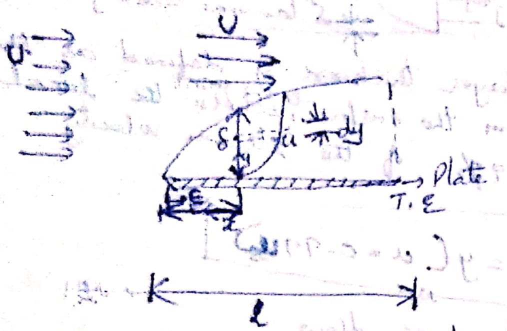
ii) for turbulent flow

According to Prandtl

$$\delta_{Tur} = \frac{0.377x}{(Re)^{1/5}}$$

* Displacement thickness (δ^*)

The Displacement thickness for the boundary layer is defined as the ⊥r. distance from the surface would have to move to compensate the reduction in flow rate due to boundary layer formation.



Consider an elemental strip 'x' distance from L.E.

Let δ be the thickness of boundary layer

Mass flow rate through the elemental strip = ρAU

where $\boxed{A = dy \times 1}$ → unit width of plate

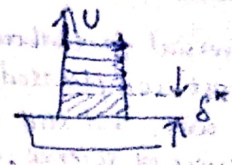
i.e.; $\boxed{\dot{m} = \rho u dy}$

Reduction in mass flow rate through the elemental strip

$= \rho (U-u) dy$

∴ Total reduction in mass flow rate

$= \int_0^\delta \rho (U-u) dy$ — ①



Let the plate is displaced at distance δ^*

Then the mass flow rate at the thickness

$\delta^* = \rho U \delta^*$ — ② (distance $y = \delta^*$)

where $\boxed{\delta^* \Rightarrow \text{displacement thickness}}$

from the definition we know that

Eqn ① = Eqn ②

$$\rho U \delta^* = \int_0^\delta \rho (U-u) dy$$

$$\boxed{\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy}$$

BASICS OF TURBULENT FLOW

The one uncontroversial fact about turbulence is that it is the most complicated kind of fluid motion. Turbulence was, and still is, one of the great unsolved mysteries of science, and it intrigued some of the best scientific minds of the day. Arnold Sommerfeld, the noted German theoretical physicist of the 1920s, once told that, for instance, that before he died he would like to understand two phenomena - quantum mechanics and turbulence. Sommerfeld died in 1924. He was somewhat nearer to an understanding of the quantum, the discovery that led to modern physics, but no closer to the meaning of turbulence.

The subject of turbulent flow is deep, extensively studied, but at the time of writing still imprecise. The basic nature of turbulence, and therefore our ability to predict its characteristics, is still an unsolved problem in classical physics. Many books have been written on turbulent flows, and many people have spent their professional lives working on the subject. Also, no pure theory of turbulent flow exists. Every analysis of turbulent-flow requires some type of empirical data in order to obtain a practical answer.

* Results for Turbulent boundary layers on a flat plates:

For incompressible flow over a flat plate, the boundary-layer thickness is given approximately by:

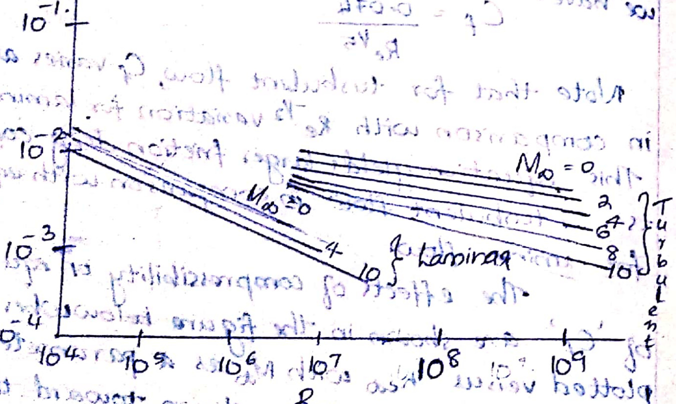
$$\delta_{tur} = \frac{0.37x}{(R_e)^{1/5}}$$

The turbulent boundary-layer thickness varies approximately as $Re^{-1/5}$ in contrast to $Re^{-1/2}$ for a laminar boundary layer. Also, turbulent value of δ grows more rapidly with distance along the surface: $\delta \alpha$ $x^{4/5}$ for a turbulent flow in contrast to $\delta \alpha x^{1/2}$ for a laminar flow. With regard to skin friction drag for incompressible turbulent flow over a flat plate, we have:

$$C_f = \frac{0.074}{Re^{1/5}}$$

Note that for turbulent flow, C_f varies as $Re^{-1/5}$ in comparison with $Re^{-1/2}$ variation for laminar flow. This equation yields larger friction drag coefficients for turbulent flow in comparison with equations for laminar flow.

The effects of compressibility on equation of 'C_f' are shown in the figure below where 'C_f' is plotted versus Re_∞ with M_∞ as a parameter. The turbulent flow results are shown toward the right of figure below, at the higher values of Reynolds number where turbulent conditions are expected to occur, and laminar flow results are shown t wards the left of the figure, at lower values of Reynolds number. This type of figure – friction drag coefficient for both laminar and turbulent flow as a function of Re on a log-log plot – is a classic picture, and it allows a ready contrast of the a types of flow. From this figure below, we can see that for the same Re_∞, turbulent skin friction is higher than laminar, also, the slopes of the turbulent curves are smaller than the slopes of the laminar curves – a graphic comparison of the $Re^{-1/5}$ variation in contrast to the laminar $Re^{-1/2}$ variation.



* Turbulent friction drag coefficient for a flat plate as a function of Reynolds and Mach numbers. Adiabatic wall $Pr = 0.75$. For contrast, some laminar results are shown.

Note that the effect of increasing M_∞ is to reduce C_f at constant Re and this effect is stronger on the turbulent flow results. Indeed, C_f for the turbulent results decreases by almost an order of magnitude (at the higher values of Re_∞) when M_∞ is increased from 0 to 10. For the laminar flow, the decrease in C_f as M_∞ is increased though the same Mach number range is far less pronounced.

* Reference Temperature Method for Turbulent flow:

With the reference temperature 'T^*' given by equation:

$$\frac{T^*}{T_e} = 1 + 0.032 M_e^2 + 0.58\left(\frac{T_w}{T_e} - 1\right).$$

the incompressible turbulent flat plate result for C_f is given by equation:

$$C_f = \frac{0.074}{Re_c^{1/5}}$$

can be modified for compressible turbulent flow as:

$$C_f^* = \frac{0.074}{(Re_x^*)^{1/5}}$$

where

$$C_f^* = \frac{D_f}{\frac{1}{2}\rho^* u_e^2 S}$$

* The Meador - Smart Reference Temperature Method for Turbulent flow:

The method developed recently by Meador and Smart gives a reference temperature equa on for turbulent flow slightly different than that for laminar flow. For a turbulent flow, the equation is:

$$\frac{T^{*}}{T_e} = 0.5\left(1+\frac{T_w}{T_e}\right) + 0.16r\left(\frac{\gamma-1}{2}\right)M_e^2$$

They also give a local turbulent skin friction coefficient for incompressible flow as:

$$C_f = \frac{\tau_w}{\frac{1}{2}\rho_e u_e^2} = \frac{0.02296}{(Re_x)^{0.139}}$$

When integrated over the entire plate of length 'c', this gives for the net skin-friction drag coefficient:

$$C_f = \frac{D_f}{\frac{1}{2}\rho_\infty V_\infty^2 S} = \frac{0.02667}{(Re_c)^{0.139}}$$

* Prediction of Airfoil Drag:

The prediction of Airfoil drag is one of the most important aspects of aerodynamics. In contrast to laminar flow there are no exact analytical solutions for turbulent flow. The analysis of any turbulent flow requires some amount of empirical data. All analyses of turbulent flow are approximate.

The analysis of the turbulent boundary layer over a flat plate is no exception.

Boundary layer thickness for incompressible turbulent flow over a flat plate is:

$$\delta = \frac{0.37x}{Re_x^{1/5}}$$

$$C_f = \frac{0.074}{Re_c^{1/5}}$$

where C_f is the skin friction drag coefficient for a turbulent flow.

We emphasize again that the above equations are only approximate results, and they represent only a set of results among a myriad of different turbulent flow analyses for the flat plate boundary layer. In contrast to the inverse square root variation with Reynolds number for laminar flow, the turbulent flow results show an inverse fifth root variation with Reynolds number.

$$C_f = \frac{1.328}{\sqrt{Re_c}} \quad \text{(laminar incompressible)}$$

For compressible flow, the numerators of these equations are no longer constants, but rather can be viewed as functions of Mach number, the ratio of wall temperature to the temperature at the outer edge of the boundary layer, T_w/T_e, and Prandtl number, Pr. The Prandtl number is defined as $Pr = \mu C_p/k$ where μ, C_p, and k are the viscosity coefficient, specific heat at constant pressure and thermal conductivity respectively.

For compressible turbulent flow we have approximately:

$$C_f = \frac{G_1(M_e, Pr, T_w/T_e)}{(Re_c)^{1/5}}$$

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For compressible laminar flow:

$$C_f = \frac{F(M_e, P_r, T_w/T_e)}{\sqrt{Re_c}}$$

The most important phenomena to observe in this figure is that C_f decreases as M_∞ increases and, that the decrease is more dramatic for a turbulent boundary layer than for a laminar boundary layer. For a given set of values of M_e, P_r and T_w/T_e, the numerical value of the numerators in the above equations are obtained from numerical solutions of the boundary layer equations.

In addition to the skin-friction drag, a supersonic airfoil also experiences supersonic wave drag. The source of wave drag is the pressure distribution exerted over the airfoil surface and is a result of the shock-wave and expansion-wave pattern in the flow over the airfoil. The source of skin-friction drag is, of course, the shear stress exerted over the airfoil surface and is the result of friction in the flow. The physical mechanisms of wave drag and skin friction drag clearly are quite different.